Primary Mathematics Challenge – February 2021

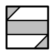
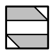
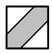
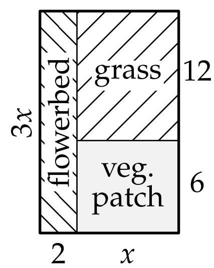
Answers and Notes

These notes provide a brief look at how the problems can be solved.

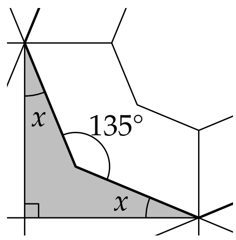
There are sometimes many ways of approaching problems, and not all can be given here.

Suggestions for further work based on some of these problems are also provided.

P1 **B** ($\frac{20}{21} = 20 \div 21 \approx 1$)   P2 **B** ($7 \times 4 - 3 = 25$)

1	**B**	6	The next time the odometer will show two digits the same is at 45 131 miles, i.e. 6 miles later.
2	**D**	15	A waiter's hourly wage, £6.81, is almost £7. To earn £100 he will have to work about $100 \div 7 \approx 15$ hours (amounting to £102.15) — 14 hours will not be enough.
3	**A**	7 minutes slow	Given that Tock's clock is 3 minutes slow, the true time is 16:34. Therefore Tick's clock is $34 - 27 = 7$ minutes slow.
4	**E**	$144\,\text{cm}^2$	The length of each side of the square is $48 \div 4 = 12$ cm. Hence the area of the square is $12 \times 12 = 144\,\text{cm}^2$.
5	**C**	4	Careful scrutiny will reveal 4 distinct tiles: , ,  and .
6	**C**	181	If we have two consecutive integers, one is odd and one is even. Hence the sum of their squares is odd. We can therefore immediately rule out 808 and 818. We can rule out 161 as it is not the same upside down. We can see that $101 = 10^2 + 1^2$ but 1 and 10 are not consecutive, so we are left with 181 which can be written as $9^2 + 10^2$.
7	**C**	7	It is clear that 8 billion, a multiple of 8, leaves a remainder of 0 when divided by 8, so 7 999 999 999 (which is 1 less) will leave a remainder of 7.
8	**B**	7	The number of diamonds Adie gets are, successively, 1, 2, 4, 8, 16, 32, ...– the powers of 2. Since $127 = 1 + 2 + 4 + 8 + 16 + 32 + 64$, she answered 7 questions correctly.
9	**E**	17	Rewriting the left-hand side $32 \times 33 \times 34$ as $(4 \times 8) \times (3 \times 11) \times (2 \times 17)$, we see that $N = 17$.
10	**C**	22	Let Patrick's number be P. We know that both $P - 500 < 0$ and $P + 501 \geq 1000$. This means that $P < 500$ and $P \geq 499$. The only possible number to satisfy both is therefore 499; the sum of its digits $4 + 9 + 9$ is 22.
11	**D**	$72\,\text{m}^2$	Given that the area of the flowerbed is equal to the area of the vegetable patch, the length of the flowerbed must be $6 \div 2 = 3$ times the width of the vegetable patch. Let x m be the width of the vegetable patch; hence the length of the flowerbed is $3x$ m. Since also the area of the flowerbed ($6x\,\text{m}^2$) is half the area of the grass.the length of the grass is 12 m. Now we have $3x = 12 + 6$, whence $x = 6$. The area of the grass is therefore $6 \times 12 = 72\,\text{m}^2$. 
12	**D**	9	Let Penny be p years old now. Then $p + 6 = 3(p - 4)$. Multiplying out the brackets here, we have $p + 6 = 3p - 12$, whence $18 = 2p$ and $p = 9$. So Penny is nine years old.
13	**B**	11	The total number of buttons is $16 + 28 + 37 = 81$. When each jar contains the same number of buttons, that number will be $81 \div 3 = 27$. In order to move the least number of buttons, 1 button from the second jar and 10 buttons from the third jar must be removed and added to the 16 buttons in the first jar. So 11 buttons altogether are moved.

14 **B** 22.5° Let the required angle be x, as in the diagram here. The interior angle of the regular octagon is $180^\circ - \frac{360^\circ}{8} = 135^\circ$, so the angle outside the octagon is $(360 - 135)^\circ = 225^\circ$. Using the angle sum of the quadrilateral, we have $2x + 90^\circ + 225^\circ = 360^\circ$, so that $x = 22.5^\circ$.



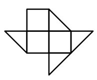
15 **C** 3 Of the five options, $13 = 13 \times 1$, $1313 = 13 \times 101$ and $131313 = 13 \times 10101$, whereas $131 = 13 \times 10 + 1$ and $13131 = 13 \times 1010 + 1$. So only three are multiples of 13.

16 **E** 50 000 000 The number of blades of grass can be calculated as $23.77 \times 100 \times 10.97 \times 100 \times 18$ which is roughly $25 \times 100 \times 10 \times 100 \times 20 = 50\,000\,000$.

17 **E** 11 Let the two numbers be j and k, with $j \geq k$. Then $(j+k)(j-k) = 21$. Given that j and k are integers, $(j+k)$ and $(j-k)$ are both factors of 21, that is 1, 3, 7 or 21 itself. If $j + k = 21$ and $j - k = 1$, then $j = 11$ and $k = 10$. Taking $(j+k)$ to be a smaller factor than 21, for instance 7, will not lead to larger values for j than 11, as shown below:

$j+k$	$j-k$	j	k
21	1	11	10
7	3	5	2
3	7	5	-2
1	21	11	-10

18 **E** 32 If we assume that in the question no cupcake is subdivided, the total number of cupcakes must be divisible by 3, 5 and 9. Such a number is 45, in which case Alice's best friend gets 15, mum 9, granny 5; in this case, mum gets 4 more than granny. It should be obvious that for mum to get 8 more than granny requires doubling the total number of cupcakes to 90. In this case, Alice is left with $90 - 2 \times 15 - 2 \times 9 - 2 \times 5 = 32$ cupcakes.

19 **D**  When the pyramid corner has been cut off, the 'truncated' cube will retain 3 square faces, but each of the other three will be reduced to half a square, that is a right-angled isosceles triangle; the base of the removed pyramid will also contribute an equilateral triangle, its edges being the diagonals of the squares cut in half. The only net that does not comprise these 7 faces is option D. The rest are working nets, as can be proved by construction!

20 **C** 190 cm Let the heights of the man, his top hat and his stilts be m, h and s centimetres respectively. We now have

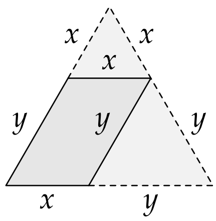
$$\begin{aligned} h + m + s &= 320 \\ h + m \qquad &= 225 \\ m + s &= 285. \end{aligned}$$

Subtracting the third equation from the first, it can be seen that $h = 320 - 285 = 35$; substituting this in the second equation gives $m = 225 - 35 - 190$, whence the man is 190 cm tall.

21 **D** 10° We can find the value of j using the angle sum of a triangle: $2j + 44 = 180$, so $j = (180 - 44) \div 2 = 68$. There are two ways to find the value of k: one will follow here and the other later in the Notes. The two unknown angles at the point where the five triangles meet are each $(180^\circ - 3 \times 44^\circ) \div 2 = 24^\circ$. Now we can find k in a similar manner to finding j, namely $k = (180 - 24) \div 2 = 78$. Hence the difference between j° and k° is $78^\circ - 68^\circ = 10^\circ$.

22 **C** 1 turn For every complete turn of the 32-cog wheel on the far left the next cog, with 16 cogs, turns twice, on account of having half the number of cogs. Two turns of the 16-cog wheel *drive* the 8-cog wheel to turn 4 times. Since the rightmost 32-cog wheel has four times as many cogs as the 8-cog wheel, the larger wheel will turn 4 quarter-turns, that is, a complete turn.

23 **E** 30 cm In order to leave a parallelogram after two cuts, each of the two cuts must be parallel to a side of the triangle, as shown here. Let the sides of the parallelogram be x and y. From the perimeter we have $2x + 2y = 20$, and so $x + y = 10$. The length of the perimeter of the triangle is $3x + 3y$, and therefore $3(x + y) = 3 \times 10 = 30$ cm.

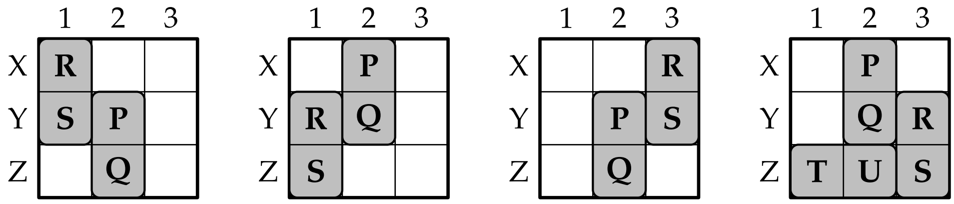


24 **A** 11 Any multiple of 12 is both a multiple of 3 and a multiple of 4. In order to be a multiple of 4, the last two digits must be a multiple of 4, so here the last two digits are '12'. In what follows we can recall that a number is divisible by 3 if and only if the sum of its digits is divisible by 3. Looking for a 3-digit multiple of 12 (_12), the extra digit must be a multiple of 3, impossible if we are allowed only 1 or 2 as digits. For 4-digit multiples (_ _12), the two extra digits must add to 3, so one 1 and one 2. For 5-digit multiples, the three extra digits must be either 111 or 222. Finally, for 6-digit multiples, we have two 1s and two 2s, as no other combination of four 1s or 2s will lead to a multiple of 3. There are 6 ways to order these four digits as shown below.

2-digits	12					
3-digits	–					
4-digits	1212	2112				
5-digits	11112	22212				
6-digits	112212	121212	122112	211212	212112	221112

Thus there are 11 multiples altogether.

25 **E** Z3 One way to approach this is to work from the four possible placements of the R/S tile, where each placement forces the subsequent placement of the P/Q tile:



The first three options leave nowhere for the T/U tile so that the T has a blank square above it. Only the last option allows this, and S goes into square Z3.

Some notes and possibilities for further problems

2 A waiter's average hourly pay of £6.81 amounts to yearly wage of £17 100. The table below shows the average yearly wage of various trades and professions*:

airline pilot	£54 000
ambulance driver	£24 000
anaesthetist	£86 900
barrister	£89 400
bricklayer	£31 800
car mechanic	£32 400
hospital staff nurse	£27 000
marketing director	£98 700
nuclear engineer	£52 500
primary school teacher	£34 500
train driver	£48 500
waiter	£17 100

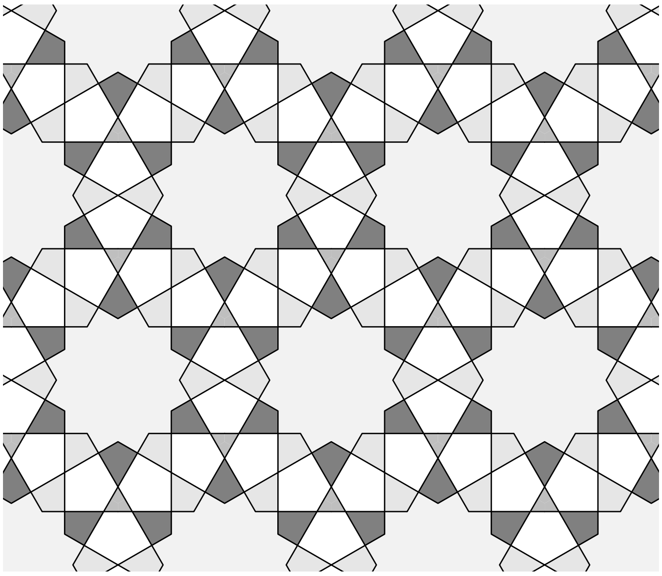
Perhaps pupils could plot these wages as a bar-chart.

*Data from https://uk.jobted.com/salary

5 The pattern in the question is taken from the excellent booklet *Pattern: its structure and geometry*, by Richard Padwick and Trevor Walker, published by Ceolfrith Press, 1977 (ISBN: 978-0904461237). Pupils may like to design their own composite using a limited variety of tiles

6 There is a well-known result, proposed by the seventeenth-century French mathematician Pierre de Fermat, that a prime number is the sum of two square numbers if and only if it is one greater than a multiple of four. Also any whole integer, N, can be written as the sum of two square numbers if and only if when written as the product of its prime factors, say $N = {p_1}^{k_1} \times {p_2}^{k_2} \times \ldots \times {p_m}^{k_m}$, there is no term where a prime p is one less than a multiple of four and its power k is odd. In the question, 161 cannot be written as the sum of two squares since $161 = 7^1 \times 23^1$, and both 7 and 23 are 1 less than multiples of four.

14 Islamic art arises from the religious interpretation that precludes the depiction of animals, including human beings. In consequence, much Islamic art instead incorporates flowers or calligraphy, or to a large extent purely geometric designs; these are often based on squares, hexagons and dodecagons, octagons, pentagons and decagons. The design below is based upon one found in the Great Mosque of Herat in north-western Afghanistan.



16 The most common truncated polyhedron is, of course, the truncated icosahedron, the standard basis for a football. Its faces comprise 12 regular pentagons and 20 regular hexagons, even though UK road signs which indicate a nearby football stadium show it as a pattern of just hexagons.

22 If, in general, a sequence of interlocking cogwheels A, B, C and D, has respectively a cogs, b cogs, c cogs and d cogs, then in one turn of cogwheel A, cogwheel B will turn through $\frac{a}{b}$ turns. Likewise, in one turn of cogwheel B, cogwheel C will turn through $\frac{b}{c}$ turns, and so on. As a result, in one turn of cogwheel A, cogwheel D will turn through $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d}$ turns. This can be simplified to $\frac{a}{d}$ turns, meaning that the number of cogs of any 'inner' cogwheel is entirely immaterial to the result — which is perhaps somewhat counter-intuitive and surprising.

24 Pupils might like to ponder whether it is true that there exist *infinitely* many multiples of any number only using the digits of that number.